This is an amendment to 6.30.2 NMAC, Section 14. The amendment replaces Section 14 (CONTENT STANDARDS: MATHEMATICS) in its entirety.

6.30.2.14   CONTENT STANDARDS: MATHEMATICS
          A.   K-4 MATHEMATICS
                    (1)   Strand: NUMBER AND OPERATIONS - Standard: Students will understand numerical concepts and mathematical operations.
                              (a)   K-4 Benchmark: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
                                        (i)   Grade K Performance Standards: Demonstrate an understanding of the place-value structure of the base-ten number system: count with understanding and recognize "how many" in sets of objects up to 20; read and write whole numbers up to 20; compare and order whole numbers up to 20; connect numerals to the quantities they represent using various physical models; use an organized counting method to keep track of quantities while counting (one-to-one correspondence) (e.g., touch object once and only once as counting a set); order sets of objects and numbers from least to most or most to least.
                                        (ii)   Grade 1 Performance Standards: Demonstrate an understanding of the place-value structure of the base-ten number system: read, write, model, and sequence whole numbers up to 100 (including filling in missing numbers in a sequence); count with understanding and recognize "how many" in sets of objects up to 50; count orally by 2s to 20 and by 5s and 10s to 100; count orally backward from 100; compare and order numbers up to 100; decompose and recombine numbers using manipulatives (e.g., by breaking numbers apart and recombining) to create and construct equivalent representations for the same number (e.g., 10 = 3 + 7 or 1 + 2 + 7 or 3 + 2 + 5); group objects by 10s and 1s to explore place value (e.g., 24 equals two tens and four ones); use ordinal numbers (e.g., what position?) and cardinal numbers (e.g., how many?) appropriately; connect number words and numbers to the quantities they represent.
                                        (iii)   Grade 2 Performance Standards: a) Understand the relationship between numbers, quantities, and place value in whole numbers up to 1,000 and develop flexible ways of thinking about numbers: use multiple models to explore place value and the base-ten number system; represent whole numbers and use them in flexible ways including decomposing and recombining numbers and see their relationships (e.g., 3 is one less than 4, one more than 2, two less than 5); identify whether a set of objects has an odd or even number of elements; compare and order numbers using a variety of terms (e.g., tens, less than, odd numbers); apply strategies for computation utilizing an understanding of place value (e.g., 48 + 25 would be 40 + 20 is 60, 8 + 5 is 13, 60 + 13 is 73); and b) Apply counting skills and number sense through meaningful activities: count and recognize "how many" in sets of objects up to 1,000; count forward and backward from given numbers to 1,000; connect number words and numerals to the quantities they represent using physical models and other representations (e.g., 23 can be twenty-three 1s, one 10 and thirteen 1s, or two 10s and three 1s); model how many parts make a whole using equal fractional parts (e.g., 1/2, 1/3, 1/4, and 1/6 as equal parts of a whole).
                                        (iv)   Grade 3 Performance Standards: a) Exhibit an understanding of the place-value structure of the base-ten number system by: reading, modeling, writing, and interpreting whole numbers up to 10,000; comparing and ordering numbers up to 1,000; recognizing the position of a given number in the base-ten number system and its relationship to benchmark numbers such as 10, 50, 100, 500; b) Use whole numbers by using a variety of contexts and models (e.g., exploring the size of 1,000 by skip-counting to 1,000 using hundred charts or strips 10 or 100 centimeters long); c) Identify some representations for some numbers and generate them by decomposing and recombining numbers (e.g., 853 = 8 x 100 + 5 x 10 + 3; 85 x 10 + 3 = 853; 853 = 900 – 50 + 3); d) Identify the relationship among commonly encountered factors and multiples (e.g., factor pairs of 12 are 1 x 12, 2 x 6, 3 x 4; multiples of 12 are 12, 24, 36); e) Use visual models and other strategies to recognize and generate equivalents of commonly used fractions and mixed numbers (e.g., halves, thirds, fourths, sixths, eighths, and tenths); f) Demonstrate an understanding of fractions as parts of unit wholes, parts of a collection or set, and as locations on a number line; and g) Use common fractions for measuring and money (e.g., using fractions and decimals as representations of the same concept, such as half of a dollar = 50 cents).
                                        (v)   Grade 4 Performance Standards: a) Exhibit an understanding of the place-value structure of the base-ten number system by reading, modeling, writing, and interpreting whole numbers up to 100,000; compare and order the numbers: recognize equivalent representations for the same number and generate them by decomposing and combining numbers (e.g., 853 = 8 x 100 + 5 x 10 + 3; 853 = 85 x 10 + 3; 853 = 900 – 50 + 3); identify the numbers less than 0 by extending the number line and using negative numbers through familiar applications (e.g., temperature, money); b) Identify fractions as parts of unit wholes, as parts of groups, and as locations on number lines: use visual models and other strategies to compare and order commonly used fractions;

use models to show how whole numbers and decimals (to the hundredths place) relate to simple fractions (e.g., 1/2, 5/10, 0.5); identify different interpretations of fractions (division of whole numbers by whole numbers, ratio, equivalence, ordering of fractions, parts of a whole or parts of a set); c) Add and subtract fractions with common and uncommon denominators using a variety of strategies (e.g., manipulatives, numbers, pictures): recognize and generate equivalent decimal forms of commonly used fractions (e.g., halves, quarters, tenths, fifths); identify the numbers less than 0 by extending the number line and using negative numbers through familiar applications (e.g., temperature, money); and d) Recognize classes of numbers (e.g., odd, even, factors, multiples, square numbers) and apply these concepts in problem-solving situations.

     (b)   K-4 Benchmark: Understand the meaning of operations and how they relate to one another.

        (i)   Grade K Performance Standards: a) Represent numbers using pictures, objects, or numerals; b) Use concrete objects to solve simple addition and subtraction story problems (e.g., oral not written);

        (ii)   Grade 1 Performance Standards: a) Use a variety of models to demonstrate an understanding of addition and subtraction of whole numbers; b) Solve addition and subtraction problems with one- and two-digit numbers (e.g., 5 + 58 = ?); c) Find the sum of three one-digit numbers to the sum of 15; d) Understand and use the inverse relationship between addition and subtraction to solve problems and check solutions (e.g., 8 + 6 = 14 is related to 14 – 6 = 8); e) Use concrete materials to investigate situations that relate to multiplication and division (e.g., equal groupings of objects, sharing equally); and f) Given simple story problems, explain verbally how to select and use appropriate operations.

        (iii)   Grade 2 Performance Standards: a) Find the sum of two whole numbers up to three digits long (e.g., 235 + 476 = ?; 564 – 273 = ?); b) Find the difference of two whole numbers up to three digits long; c) Understand and use the inverse relationships between addition and subtraction to solve problems and check solutions (28 + 31 = 59; therefore, 59 - 31 = 28); and d) Identify and describe situations that require multiplication and division and develop strategies to solve problems for repeated joining of groups and partitioning into equal subgroups or shares (e.g., repeated addition and subtraction, counting by multiples, equal sharing).

        (iv)   Grade 3 Performance Standards: a) Use a variety of models to show an understanding of multiplication and division of whole numbers (e.g., charts, arrays, diagrams, and physical models [i.e., modeling multiplication with a variety of pictures, diagrams, and concrete tools to help students learn what the factors and products represent in various contexts]); b) Find the sum or difference of two whole numbers between 0 and 10,000; c) Solve simple multiplication and division problems (e.g., 135 ÷ 5 = ?); d) Identify how the number of groups and the number of items in each group equal a product; e) Demonstrate the effects of multiplying and dividing on whole numbers (e.g., to find the total number of legs on 12 cats, 4 represents the number of each [cat] unit, so 12 x 4 = 48 [leg] units; f) Identify and use relationship between multiplication and division (e.g., division is the inverse of multiplication) to solve problems; and g) Select and use operations (e.g., addition, multiplication, subtraction, division) to solve problems.

        (v)   Grade 4 Performance Standards: a) Demonstrate an understanding of and the ability to use: standard algorithms for the addition and subtraction of multi-digit numbers; standard algorithms for multiplying a multi-digit number by a two-digit number and for dividing a multi-digit number by a one-digit number; b) Select and use appropriate operations (addition, subtraction, multiplication, and division) to solve problems; c) Extend the uses of whole numbers to the addition and subtraction of simple decimals (positive numbers to two places); d) Demonstrate commutative, associative, identity, and zero properties of operations on whole numbers (e.g., 37 x 46 = 46 x 37 and (6 x 2) x 5 = 6 x (2 x 5)); and e) Demonstrate the concept of distributivity of multiplication over addition and subtraction (e.g., 7 x 28 is equivalent to (7 x 20) + (7 x 8) or (7 x 30) - (7 x 2)).

     (c)   K-4 Benchmark: Compute fluently and make reasonable estimates.

        (i)   Grade K Performance Standards: Estimate quantities of objects up to 20.

        (ii)   Grade 1 Performance Standards: a) Use strategies for whole-number computation, with a focus on addition and subtraction (e.g., counting on or counting back, doubles, sums that make 10, direct modeling with pictures or objects, numerical reasoning based on number combinations and relationships); b) Demonstrate a variety of methods to compute (e.g., objects, mental computation, paper and pencil, and estimation); c) Perform addition and subtraction with whole number combinations; and d) Use and explain estimation strategies to determine the reasonableness of answers involving addition and subtraction.

        (iii)   Grade 2 Performance Standards: a) Use and explain strategies for addition and subtraction of multi-digit whole numbers; b) Model and solve problems representing adding and subtracting amounts of money using dollars and coins; c) Use addition combinations (addends through 10) and related subtraction combinations, and develop strategies for computing based on number sense (e.g., 25 + 37: Take 3 from the 25 and use it to turn 37 into 40; then add 40 and 22 to get 62); d) Select and use a variety of appropriate strategies and methods to compute (e.g., objects, mental computation, estimation, paper and pencil); and e) Skip-

count by 2, 5, and 10 to develop multiplicative reasoning and notational representations (e.g., 5, 10, 15, 20; 4 x 5 = 20; four groups of 5 equal 20).

               (iv)   Grade 3 Performance Standards: a) Choose computational methods based on understanding the base-ten number system, properties of multiplication and division, and number relationships; b) Use strategies (e.g., 6 x 8 is double 3 x 8) to become fluent with the multiplication pairs up to 10 x 10; c) Compute with basic number combinations (e.g., multiplication pairs up to 10 x 10 and their division counterparts); and d) Demonstrate reasonable estimation strategies for measurement, computation, and problem solving.

               (v)   Grade 4 Performance Standards: a) Demonstrate multiplication combinations through 12 x 12 and related division facts, and use them to solve problems mentally and compute related problems (e.g., 4 x 5 is related to 40 x 50, 400 x 5, and 40 x 500); b) Add, subtract, and multiply up to two double-digits accurately and efficiently; c) Use a variety of strategies (e.g., rounding and regrouping) to estimate the results of whole number computations and judge the reasonableness of the answers; and d) Use strategies to estimate computations involving fractions and decimals.

         (2)   Strand: ALGEBRA - Standard: Students will understand algebraic concepts and applications.

           (a)   K-4 Benchmark: Understand patterns, relations, and functions.

               (i)   Grade K Performance Standards: a) Identify the attributes of objects (e.g., the ability to identify attributes is a foundational skill for sorting and classifying); b) Sort, classify, and order objects by size, number, and other properties; and c) Recognize, reproduce, describe, extend, and create repeating patterns (e.g., color, shape, size, sound, movement, simple numbers).

               (ii)   Grade 1 Performance Standards: a) Recognize, reproduce, describe, extend, and create repeating patterns (e.g., color, shape, size, sound, movement, simple numbers) and translate from one representation to another (e.g., red, red, blue, blue to step, step, clap, clap); b) Skip-count on a hundreds chart (e.g., by 2s up to 20 and 5s and 10s up to 100) to identify, describe, and predict number patterns; and c) Identify number patterns on the hundreds chart.

               (iii)   Grade 2 Performance Standards: a) Recognize, reproduce, describe, extend, and create repeating and growing patterns, and translate from one representation to another; b) Skip-count using calculators or a hundreds chart to identify, describe, predict, and make generalizations about number patterns to differentiate rote counting versus the meaning of the numbers; c) Construct and solve open sentences that have variables (e.g., 10 = ? + 7); and d) Relate everyday problem situations to number sentences involving addition and subtraction (e.g., 25 students are going to the store. Five students can ride in a car. How many cars will be needed?).

               (iv)   Grade 3 Performance Standards: a) Represent relationships of quantities in the form of mathematical expressions, equations, or inequalities; b) Solve problems involving numeric equations; c) Select appropriate operational and relational symbols to make an expression true (e.g., "If 4 ? 3 = 12, what operational symbol goes in the box?"); d) Use models of feet and inches to express simple unit conversions in symbolic form (e.g., 36 inches = ? feet x 12) that develop conceptual understanding versus procedural skills; e) Recognize and use the commutative property of multiplication (e.g., if 5 x 7 = 35, then what is 7 x 5?); f) Create, describe, and extend numeric and geometric patterns including multiplication patterns; and g) Represent simple functional relationships: solve simple problems involving a functional relationship between two quantities (e.g., find the total cost of multiple items given the cost per unit); extend and recognize a linear pattern by its rules (e.g., the number of legs on a given number of horses may be calculated by counting by 4s, by multiplying the number of horses by 4, or through the use of tables).

               (v)   Grade 4 Performance Standards: a) Represent and analyze patterns and simple functions using words, tables, and graphs; b) Create and describe numeric and geometric patterns including multiplication and division patterns; c) Express mathematical relationships using equations; and d) Use and interpret variables, mathematical symbols, and properties to write and simplify expressions and sentences: use letters, boxes, or other symbols to stand for any number in simple expressions or equations (e.g., demonstrate an understanding of the concept of a variable); interpret and evaluate mathematical expressions using parentheses; use and interpret formulas (e.g., Area = Length x Width or A = L x W) to answer questions about quantities and their relationships.

           (b)   K-4 Benchmark: Represent and analyze mathematical situations and structures using algebraic symbols.

               (i)   Grade K Performance Standards: Use concrete, pictorial, and verbal representation to develop an understanding of invented and conventional symbols.

               (ii)   Grade 1 Performance Standards: a) Write number sentences that use concrete objects, pictorial, and verbal representations to express mathematical situations using invented and conventional symbols (e.g., +, -, =); b) Demonstrate and describe the concept of equal (e.g., using objects, balance scales); and c) Solve open number sentences that have variables representing numbers up to 10 (e.g., 10 = ? + 2).

        (iii)   Grade 2 Performance Standards: a) Use mathematical language to describe a variety of representations and mathematical ideas and situations; b) Explain the concept of equal (e.g., quantities on both sides of equation are the same) by using objects or giving examples; c) Construct and solve open number sentences that have variables representing numbers up to 20 (e.g., 20 = ? + 6); and d) Use objects, words, and symbols to explain the concept of addition.

        (iv)   Grade 3 Performance Standards: a) Determine the value of variables in missing part problems (e.g., 139 + ? = 189); b) Recognize and use the commutative and associative properties of addition and multiplication (e.g., "If 5 x 7 = 35, then what is 7 x 5? And if 5 x 7 x 3 = 105, then what is 7 x 3 x 5?"); and c) Explore the ways that commutative, distributive, identity, and zero properties are useful in computing with numbers.

        (v)   Grade 4 Performance Standards: a) Identify symbols and letters that represent the concept of a variable as an unknown quantity; b) Explore the uses of properties (commutative, distributive, associative) in the computation of whole numbers; c) Express mathematical relationships using equations; d) Determine the value of variables in simple equations (e.g., 80 x 15 = 40 x ?); and e) Develop simple formulas in exploring quantities and their relationships (e.g., A = L x W).

        (c)   K-4 Benchmark: Use mathematical models to represent and understand quantitative relationships.

        (i)   Grade K Performance Standards: Model situations that involve whole numbers using objects or pictures.

        (ii)   Grade 1 Performance Standards: a) Represent equivalent forms of the same number through the use of physical models, diagrams, and number expressions to 20 (e.g., 3 + 5 = 8, 2 + 6 = 8); and b) Describe situations that involve addition and subtraction of whole numbers including objects, pictures, and symbols (e.g., Robert has four apples, Maria has five more).

        (iii)   Grade 2 Performance Standards: a) Model situations of addition and subtraction of whole numbers using objects, pictures, and symbols; b) Solve problems related to trading (e.g., coin trading, measurement trading); and c) Solve addition and subtraction problems by using data from simple charts, picture graphs, and number sentences.

        (iv)   Grade 3 Performance Standards: a) Model problem situations with objects and use representations such as pictures, graphs, tables, and equations to draw conclusions; b) Solve problems involving proportional relationships including unit pricing (e.g., four apples cost 80 cents; therefore, one apple costs 20 cents); c) Describe relationships of quantities in the form of mathematical expressions, equations, or inequalities; and d) Select appropriate operational and relational symbols to make an expression true (e.g., "If 4 ? 3 = 12, what operational symbol goes in the box?").

        (v)   Grade 4 Performance Standards: a) Solve problems involving proportional relationships (including unit pricing and map interpretations; e.g., one inch = five miles; therefore, five inches = ? miles); b) Model problem situations and use graphs, tables, pictures, and equations to draw conclusions (e.g., different patterns of change); and c) Use and interpret formulas (e.g., Area = Length x Width or A = L x W) to answer questions about quantities and their relationships.

        (d)   K-4 Benchmark: Analyze changes in various contexts.

        (i)   Grade K Performance Standards: Verbally describe changes in various contexts (e.g., plants or animals growing over time).

        (ii)   Grade 1 Performance Standards: Describe qualitative change (e.g., a student growing taller, trees getting bigger, ice melting).

        (iii)   Grade 2 Performance Standards: Describe quantitative change (e.g., a student growing two inches in one year, water heating up to boil).

        (iv)   Grade 3 Performance Standards: Demonstrate how change in one variable can relate to a change in a second variable (e.g., input-output machines, data tables).

        (v)   Grade 4 Performance Standards: a) Identify and describe situations with constant or varying rates of change and compare them; b) Determine how a change in one variable relates to a change in a second variable (e.g., data tables, input-output machines); c) Find and analyze patterns using data tables (e.g., T tables); and d) Demonstrate and describe varying rates of change in relation to real-world situations (e.g., plant growth, students' heights).

        (3)   Strand: GEOMETRY - Standard: Students will understand geometric concepts and applications.

        (a)   K-4 Benchmark: Analyze characteristics and properties of two- and three-dimension geometric shapes and develop mathematical arguments about geometric relationships.

        (i)   Grade K Performance Standards: Identify common objects in their environments and describe their geometric features: describe, identify, model, and draw common geometric objects (e.g., circle,

triangle, square, rectangle, cube, sphere, cone); compare familiar plane and solid objects by common attributes (e.g., shape, size, number of corners).

    (ii)   Grade 1 Performance Standards: Identify common geometric figures and classify them by common attributes: recognize, name, build, and draw both polygonal (up to six sides) and curved shapes; sort two- and three-dimensional shapes into categories based on common attributes; use the attributes of shapes to analyze and identify examples and non-examples of geometric shapes; participate in discussions comparing, identifying, and analyzing attributes to develop the vocabulary needed to describe two- and three-dimensional geometric shapes and their attributes (e.g., sides, corners, edges, faces).

    (iii)   Grade 2 Performance Standards: Identify and describe the attributes of common figures in a plane and common objects in space: sort, describe, and analyze plane and solid geometric shapes (e.g., circle, triangle, square, rectangle, sphere, pyramid, cube, rectangular prism) based on various attributes (e.g., faces, edges, and corners); put shapes together and take them apart to form other shapes (e.g., two congruent right triangles can be arranged to form a rectangle); explore lines of symmetry in two-dimensional shapes.

    (iv)   Grade 3 Performance Standards: Describe and compare the attributes of plane and solid geometric figures to show relationships and solve problems: identify, describe, and classify polygons (e.g., pentagons, hexagons, and octagons); identify lines of symmetry in two-dimensional shapes; explore attributes of quadrilaterals (e.g., parallel and perpendicular sides for the parallelogram, right angles for the rectangle, equal sides and right angles for the square); identify right angles; identify, describe, and classify common three-dimensional geometric objects (e.g., cube, rectangular solid, sphere, prism, pyramid, cone, cylinder).

    (v)   Grade 4 Performance Standards: a) Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes: build, draw, create, and describe geometric objects; identify lines that are parallel or perpendicular; identify and compare congruent and similar figures; b) Classify two- and three-dimensional shapes according to their properties and develop definitions of classes like triangles and pyramids: visualize, describe, and make models of geometric solids in terms of the number of faces, edges, and vertices; interpret two-dimensional representations of three-dimensional objects; and c) Make and test conjectures about geometric properties and relationships and develop logical arguments to justify conclusions.

    (b)   K-4 Benchmark: Specify locations and describe spatial relationships using coordinate geometry and other representational systems.

    (i)   Grade K Performance Standards: a) Follow simple directions to find a specific location in space; and b) Use spatial vocabulary (e.g., left, right, above, below) to describe relative position.

    (ii)   Grade 1 Performance Standards: Participate in group and individual activities based on the concepts of space and location: describe direction, location, space, and shape (e.g., left, right, over, under, near, far, between); visualize, describe, and record directions for navigating from one location to another to develop the vocabulary needed to describe direction, distance, location, and representation; use materials to create representations of the surrounding environment (e.g., three-dimensional models, maps of the classroom); develop estimates and measure distances using nonstandard measurements.

    (iii)   Grade 2 Performance Standards: a) Find and name locations with simple relationships like "near to" and apply ideas about relative position; b) Describe, name, and interpret direction in navigating space and apply ideas about direction and distance; c) Use maps to locate points and navigate through mazes or maps; d) Visualize, justify, and create paths using landmarks, space, shapes, and descriptive language; and e) Make and draw rectangular arrays of squares.

    (iv)   Grade 3 Performance Standards: a) Describe location and movement using common language and geometric vocabulary (e.g., directions from classroom to gym); b) Use ordered pairs to graph, locate specific points, create paths, and measure distances within a coordinate grid system; and c) Use a two-dimensional grid system (e.g., a map) to locate positions representing actual places.

    (v)   Grade 4 Performance Standards: a) Describe location and movement using common language and geometric vocabulary; b) Use ordered pairs to graph, locate, identify points, and describe paths in the first quadrant of the coordinate plane; and c)   Use a variety of methods for measuring distances between locations on a grid.

    (c)   K-4 Benchmark: Apply transformation and use symmetry to analyze mathematical situations.

    (i)   Grade K Performance Standards: a) Use manipulatives (e.g., puzzles, tangrams, blocks) to demonstrate rotation (i.e., turns), translations (i.e., slides), and reflection (i.e., flips); and b) Investigate the symmetry of two-dimensional shapes (e.g., by folding or cutting paper, using mirrors).

(ii)   Grade 1 Performance Standards: a) Predict the results of changing a shape's position or orientation by using rotation (i.e., turns), reflection (i.e., flips), and translations (i.e., slides); b) Create simple symmetrical shapes and pictures; and c) Recognize and describe the symmetric characteristics of designs (e.g., geometric designs made with pattern blocks).

(iii)   Grade 2 Performance Standards: a) Use systematic thinking to solve geometric puzzles (e.g., pentominoes); and b) Use materials to investigate rotational and line symmetry and create shapes that have symmetry.

(iv)   Grade 3 Performance Standards: a) Predict and describe the results of sliding, flipping, and turning two-dimensional shapes; and b) Identify and describe the line of symmetry in two- and three-dimensional shapes.

(v)   Grade 4 Performance Standards: a) Create and describe rotational designs using language of transformational symmetry; and b) Describe a motion or set of motions that will show that two shapes are congruent.

(d)   K-4 Benchmark: Use visualization, spatial reasoning, and geometric modeling to solve problems.

(i)   Grade K Performance Standards: a) Describe how to get from one location to another (e.g., how to get to the library); and b) Find and describe geometric shapes in nature or architecture.

(ii)   Grade 1 Performance Standards: a) Use combinations of shapes to make a new shape to demonstrate relationships between shapes (e.g., a hexagon can be made from six triangles); b) Create three-dimensional shapes based on two-dimensional representations; c) Participate in activities to develop mental visualization and spatial memory (e.g., "quick image" activities that require students to recall or reproduce a configuration of dots on a card or to determine the number of dots without counting); d) Describe how to get from one location to another by visualizing the landmarks along the route; and e) Identify structures from different views or match views of the same structure portrayed from different perspectives.

(iii)   Grade 2 Performance Standards: a) Demonstrate relationships of different attributes with concrete materials (e.g., change one characteristic of a shape while preserving others such as increasing number of sides while perimeter stays the same); b) Select and use visualization skills to create mental images of geometric shapes; c) Describe geometric shapes and structures from different perspectives; d) Relate geometric ideas to numbers (e.g., seeing rows in array as a model of repeated addition); and e) Recognize geometric shapes and structures in the environment and specify their location.

(iv)   Grade 3 Performance Standards: a) Visualize, build, and draw geometric objects; b) Create and describe mental images of objects, patterns, and paths; c) Recognize geometric shapes and structures (e.g., in the environment); d) Use geometric models to solve problems in other areas of mathematics (e.g., using arrays as models of multiplication or area); e) Identify and build three-dimensional objects from two-dimensional representations of that object; f) Investigate two-dimensional representations of three-dimensional shapes; and g) Explore geometric ideas and relationships as they apply to other disciplines and to problems that arise in the classroom or in everyday life.

(v)   Grade 4 Performance Standards: a) Develop and use mental images of geometric shapes to solve problems (e.g., represent three-dimensional shapes in two dimensions); b) Use geometric models such as number lines, arrays, and computer simulations to investigate number relationships (e.g., patterns); and c) Explore relationships involving perimeter and area: measure area of rectangular shapes and use appropriate units; recognize that area can have the same perimeter but different areas and vice versa; use models and formulas to solve problems involving perimeter and area of rectangles and squares (e.g., arrays).

(4)   Strand: MEASUREMENT - Standard: Students will understand measurement systems and applications.

(a)   K-4 Benchmark: Understand measurable attributes of objects and the units, systems, and process of measurement.

(i)   Grade K Performance Standards: a) Describe and compare, using appropriate concepts and vocabulary, the measurable properties of length (e.g., shorter, longer, taller), volume (e.g., full, empty), weight (e.g., heavy, light), and time (e.g., before, after, morning, afternoon, days of week); b) Use tools to make predictions (e.g., using a balance scale, predicting how many cups a container will hold and then filling it to check the prediction); c) Measure using non-standard units of measurement (e.g., use pencils to measure desk top, use different lengths of rope to measure distance in classroom); and dd) Use digital and analog (face) clocks to tell time to the hour.

(ii)   Grade 1 Performance Standards: a) Develop an understanding of measurable properties (e.g., length, volume, weight, area, and time) using appropriate concepts and vocabulary: length by

measuring and estimating (e.g., longer, shorter, meter, centimeter, inch, yard); weight by measuring, estimating, and weighing (e.g., heavy [-ier], light [-er]); volume by measuring, estimating, and weighing (e.g., full, empty); area by measuring and estimating (e.g., perimeter, rectangles, squares); time by estimating (e.g., minutes, hours, days, weeks); and b) Use digital and analog (face) clocks to tell time to the half hour.

             (iii)   Grade 2 Performance Standards: a) Identify a unit of measure (e.g., nearest inch) and repeat that unit comparing it to the item being measured; b) Use direct comparison to compare and order objects according to length, mass, and area; c) Measure and compare common objects using standard and non-standard units of length; d) Find and represent the value of a collection of coins and dollars up to $5.00, using appropriate notation; e) Identify and use time intervals (e.g., hours, days, weeks, months); f) Select and use appropriate measurement tools (e.g., ruler, yardstick, meter stick); and g) Tell time to the nearest quarter hour.

             (iv)   Grade 3 Performance Standards: a) Demonstrate understanding of the need for measuring with standard units and become familiar with standard units in the U.S. customary system; b) Choose and use the appropriate units and measurement tools to quantify the properties of objects (e.g., length [ruler], width [ruler], or mass [balance scale]); c) Identify time to the nearest minute (elapsed time) and relate time to everyday events; d) Identify and use time intervals (e.g., hours, days, weeks, months, years); e) Identify properties (e.g., length, area, weight, volume) and select the appropriate type of unit for measuring each property; and f) Demonstrate understanding that measurements are approximations, investigate differences in units and their effect on precision, and consider the degree of accuracy for different situations.

             (v)   Grade 4 Performance Standards: a) Select the appropriate type of unit for measuring perimeter and size of an angle; b) Understand the need for measuring with standard units and become familiar with the standard units in customary and metric system; c) Identify the inverse relationship between the size of the units and the number of units; d) Develop formulas to determine the surface areas of rectangular solids; e) Develop, understand, and use formulas to find the area of rectangles and related triangles and parallelograms; and f) Carry out simple conversions within a system of measurement (e.g., hours to minutes, meters to centimeters).

          (b)   K-4 Benchmark: Apply appropriate techniques, tools, and formulas to determine measurements.

             (i)   Grade K Performance Standards: Explore measuring objects using a repeating non-standard unit of measurement (e.g., paper clips, cubes, etc.).

             (ii)   Grade 1 Performance Standards: a) Measure with multiple copies of units the same size (e.g., paper clips); and b) Use repetition of a single unit to measure something larger than the unit (e.g., a yardstick/meterstick to measure a room).

             (iii)   Grade 2 Performance Standards: a) Develop common referents to make comparisons and estimates of length, volume, weight, area, and time; b) Develop an understanding that different measuring tools will yield different numerical measurements of the same object (e.g., ruler, yardstick, meterstick, paper clip); and c) Estimate measurements and develop precision in measuring objects.

             (iv)   Grade 3 Performance Standards: a) Find the area of rectangles using appropriate tools (e.g., grid paper, tiles); b) Estimate measurements; c) Use appropriate standard units and tools to estimate, measure, and solve problems (e.g., length, area, weight); and d) Recognize a 90-degree angle and use it as a strategy to estimate the size of other angles.

             (v)   Grade 4 Performance Standards: a) Estimate perimeters, areas of rectangles, triangles, and irregular shapes; b) Find the area of rectangles, related triangles, and parallelograms; c) Estimate, measure, and solve problems involving length, area, mass, time, and temperature using appropriate standard units and tools; d) Identify common measurements of turns (e.g., 360 degrees in one turn, 90 degrees in a quarter-turn); e) Compute elapsed time and make and interpret schedules; and f) Use tools to measure angles (e.g., protractor, compass).

         (5)   Strand: DATA ANALYSIS AND PROBABILITY - Standard: Students will understand how to formulate questions, analyze data, and determine probabilities.

          (a)   K-4 Benchmark: Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.

             (i)   Grade K Performance Standards: Collect data about objects and events in the environment to answer simple questions (e.g., brainstorm questions about self and surroundings, collect data, and record the results using objects, pictures, and pictographs).

             (ii)   Grade 1 Performance Standards: Collect, organize, represent, and compare data by category on graphs and charts to answer simple questions: answer questions about "how" data can be gathered; gather data by interviewing, surveying, and making observations; organize data into appropriate categories by sorting based on shared properties; participate in discussions about selecting an appropriate way to display the data;

represent data using objects, pictures, tables, and simple bar graphs.

                    (iii)   Grade 2 Performance Standards: a) Collect numerical data systematically; b) Represent data by using concrete objects, pictures, tables, numbers, tallies, and graphs (e.g., pictographs); c) Pose questions about students' selves and their surroundings and gather data by interviewing, surveying, and making observations to answer the questions posed; and d) Identify patterns and explain the relationships of the units in the pattern (e.g., the number of ears on one dog, two dogs, etc., or linear numerical patterns).

                    (iv)   Grade 3 Performance Standards: a) Collect and organize data using observations, measurements, surveys, or experiments; b) Represent data using tables and graphs (e.g., line plots, bar graphs, and line graphs); and c) Conduct simple experiments by determining the number of possible outcomes and make simple predictions: identify whether events are certain, likely, unlikely, or impossible; record the outcomes for a simple event and keep track of repetitions; summarize and record the results in a clear and organized way; use the results to predict future events.

                    (v)   Grade 4 Performance Standards: a) Organize, represent, and interpret numerical and categorical data and clearly communicate findings: choose and construct representations that are appropriate for the data set; recognize the differences in representing categorical and numerical data; and b) Design investigations and represent data using tables and graphs (e.g., line plots, bar graphs, line graphs).

            (b)   K-4 Benchmark: Select and use appropriate statistical methods to analyze data.

                    (i)   Grade K Performance Standards: Describe simple data and pose questions about the data.

                    (ii)   Grade 1 Performance Standards: Analyze simple data: interpret what the graph or other representation shows; determine whether or not the data gathered helps answer the specific question that was posed; compare parts of the data (e.g., "How many students have lost none, one, two, or three teeth?") to make statements about the data as a whole (e.g., "Most students in the class have lost only two teeth.").

                    (iii)   Grade 2 Performance Standards: a) Describe and interpret data by drawing conclusions and making conjectures based on the data collected; and b) Display data in a variety of formats.

                    (iv)   Grade 3 Performance Standards: Apply and explain the uses of sampling techniques (e.g., observations, polls, tally marks) for gathering data.

                    (v)   Grade 4 Performance Standards: a) Compare and describe related data sets; b) Use the concepts of median, mode, maximum, minimum, and range and draw conclusions about a data set; and c) Use data analysis to make reasonable inferences/predictions and to develop convincing arguments from data described in a variety of formats (e.g. bar graphs, Venn diagrams, charts, tables, line graphs, and pictographs).

            (c)   K-4 Benchmark: Develop and evaluate inferences and predictions that are based on data.

                    (i)   Grade K Performance Standards: Make simple predictions.

                    (ii)   Grade 1 Performance Standards: Make conclusions based on data (e.g., whether or not other groups would reach similar conclusions based on the same data).

                    (iii)   Grade 2 Performance Standards: a) Discuss events related to students' experiences as "likely" or "unlikely" and "possible" or "certain"; b) Recognize appropriate conclusions generated from the data collected; and c) Recognize inappropriate descriptions of the data set.

                    (iv)   Grade 3 Performance Standards: Analyze data displayed in a variety of formats to make reasonable inferences and predictions, answer questions, and make decisions.

                    (v)   Grade 4 Performances Standards: a) Propose and justify conclusions and predictions based on data; and b) Develop convincing arguments from data displayed in a variety of formats.

            (d)   K-4 Benchmark: Understand and apply basic concepts of probability.

                    (i)   Grade K Performance Standards: Answer questions that relate to the possibility of familiar events happening or not.

                    (ii)   Grade 1 Performance Standards: a) Discuss the likelihood of events (based on student experiences or from books) using terminology such as "more likely", "less likely", "possible", or "certain"; and b) Observe, explore, and discuss whether some events occur more often than others (e.g., tossing two die and recording the sum after each toss to explore whether or not certain sums occur more frequently than others).

                    (iii)   Grade 2 Performance Standards: a) Investigate concepts of chance (e.g., outcomes of a simple experiment); and b) Investigate whether outcomes of a simple event are equally likely to occur.

                    (iv)   Grade 3 Performance Standards: a) Discuss the degree of likelihood of events and use terminology such as "certain," "likely," "unlikely"; b) Predict the outcomes of simple experiments (e.g., coin tossing) and test the predictions using concrete objects (e.g., coins, counters, number cubes, spinners); and c) Record the probability of a specific outcome for a simple probability situation (e.g., probability is three out of seven for choosing a black ball; 3/7).

(v)   Grade 4 Performance Standards: a) Describe events as "likely," "unlikely," or "impossible" and quantify simple probability situations: represent all possible outcomes for a simple probability situation in an organized way (e.g., tables, grids, tree diagrams); express outcomes of experimental probability situations verbally and numerically (e.g., three out of four, 3/4); and b) List all the possible combinations of objects from three sets (e.g., spinners, number of outfits from three different shirts, two skirts, and two hats).

    B.   5-8 MATHEMATICS

    (1)   Strand: NUMBER OF OPERATIONS - Standard: Students will understand numerical concepts and mathematical operations.

    (a)   5-8 Benchmark: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.

(i)   Grade 5 Performance Standards: a) Compare and order using concrete or illustrated models: whole numbers (to millions); common fractions (halves, thirds, fourths, eighths); decimals (thousandths); b) Demonstrate understanding of the magnitude of the value of numbers from thousandths to millions, including common fractions; c) Represent place value using concrete or illustrated models up to one billion (1,000,000,000); d) Interpret percents as part of a hundred (i.e., find decimal and percent equivalents for common fractions, explain how they represent the same value, and compute a given percent of a whole number); e) Identify and represent on a number line decimals, fractions, and mixed numbers; and f) Identify prime and composite numbers to 50.

(ii)   Grade 6 Performance Standards: a) Compare and order rational numbers; b) Use equivalent representations for rational numbers (e.g., integers, decimals, fractions, percents, ratios, numbers with whole-number exponents); c) Use appropriate representations of positive rational numbers in the context of real-life applications; d) Identify greatest common factor and least common multiples for a set of whole numbers; and e) Identify and represent on a number line decimals, fractions, mixed numbers, and positive and negative integers.

(iii)   Grade 7 Performance Standards: a) Determine the absolute value of rational numbers; b) Illustrate the relationships among natural (i.e., counting) numbers, whole numbers, integers, rational and irrational numbers; c) Use properties of the real-number system to explain reasoning and to formulate and solve real-world problems; d) Read, write, and compare rational numbers in scientific notation (e.g., positive and negative powers of 10) with approximate numbers using scientific notation; and e) Simplify numerical expressions using order of operations.

(iv)   Grade 8 Performance Standards: a) Sort numbers by their properties (e.g., prime, composite, square, square root); and b) Demonstrate the magnitude of rational numbers (e.g., trillions to millions).

    (b)   5-8 Benchmark: Understand the meaning of operations and how they relate to one another.

(i)   Grade 5 Performance Standards: a) Explain and perform whole number division and express remainders as a whole number or a fractional part as appropriate to the context of real-life problems; b) Add and subtract decimals; c) Add and subtract fractions and mixed numbers without regrouping and express answers in simplest form; d) Find the factors and multiples of whole numbers; e) Use arithmetic operations and inverse relationships to represent and solve real-world problems; f) Identify and represent on a number line decimals, fractions, and mixed numbers; g) Demonstrate proficiency with division, including one- and two-digit divisors; h) Solve simple problems involving the addition and subtraction of fractions and mixed numbers; and i) Represent and use fractions and decimals in equivalent forms.

(ii)   Grade 6 Performance Standards: a) Calculate multiplication and division problems using contextual situations; b) Factor a whole number into a product of its primes; c) Demonstrate the relationship and equivalency among ratios and percents; d) Use proportions to solve problems; e) Explain and perform: whole number division and express remainders as decimals or appropriately in the context of the problem; addition, subtraction, multiplication, and division with decimals; addition and subtraction with integers; addition, subtraction, and multiplication with fractions and mixed numerals; and f) Determine the least common multiple and the greatest common divisor of whole numbers and use them to solve problems with fractions.

(iii)   Grade 7 Performance Standards: a) Add, subtract, multiply, and divide rational numbers (e.g., integers, fractions, terminating decimals) and take positive rational numbers to whole-number powers; b) Convert terminating decimals into reduced fractions; c) Calculate given percentages of quantities and use them to solve problems (e.g., discounts of sales, interest earned, tips, markups, commission, profit, simple interest); d) Add and subtract fractions with unlike denominators; e) Multiply, divide, and simplify rational numbers by using exponent rules; f) Understand the meaning of the absolute value of a number: interpret the absolute value as the distance of the number from zero on a number line; determine the absolute value of real numbers; g) Find square roots of perfect whole-number squares; h) Simplify and evaluate positive rational numbers raised to positive whole number powers; and i) Solve addition, subtraction, multiplication, and division problems that use positive and negative integers and combinations of these operations.

(iv)   Grade 8 Performance Standards: a) Use real number properties (e.g., commutative, associative, distributive) to perform various computational procedures; b) Perform arithmetic operations and their inverses (e.g., addition/subtraction, multiplication/division, square roots of perfect squares, cube roots of perfect cubes) on real numbers; and c) Find roots of real numbers using calculators.

(c)   5-8 Benchmark: Compute fluently and make reasonable estimates.

(i)   Grade 5 Performance Standards: a) Add, subtract, multiply, and divide whole numbers; b) Add and subtract decimals; c) Use estimation strategies to verify the reasonableness of calculated results; d) Explain how the estimation strategy impacts the result; e) Relate the basic arithmetic operations to one another (e.g., multiplication and division are inverse operations); f) Simplify numerical expressions using order of operations; and g) Recognize and explain the differences between exact and approximate values.

(ii)   Grade 6 Performance Standards: a) Estimate quantities involving rational numbers using various estimations; b) Use estimates to check reasonableness of results and make predictions in situations involving rational numbers; c) Determine if a problem situation calls for an exact or approximate answer and perform the appropriate computation; d) Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line; e) Convert fractions to decimals and percents and use these representations in estimations, computations, and applications; f) Interpret and use ratios in different contexts; and g) Compute and perform multiplication and division of fractions and decimals and apply these procedures to solving problems.

(iii)   Grade 7 Performance Standards: a) Use estimation to check reasonableness of results, and use this information to make predictions in situations involving rational numbers, pi, and simple algebraic equations; b) Convert fractions to decimals and percents and use these representations in estimations, computations, and applications; c) Read, write, and compare rational numbers in scientific notation (e.g., positive and negative powers of 10) with approximate numbers using scientific notation; d) Calculate the percentage of increases and decreases of a quantity; e) Add and subtract fractions with unlike denominators; and f) Use the inverse relationship between rising to a power and extracting the root of a perfect square integer.

(iv)   Grade 8 Performance Standards: a) Formulate algebraic expressions that include real numbers to describe and solve real-world problems; b) Use a variety of computational methods to estimate quantities involving real numbers; c) Differentiate between rational and irrational numbers; d) Use real number properties to perform various computational procedures and explain how they were used; e) Perform and explain computations with rational numbers, pi, and first-degree algebraic expressions in one variable in a variety of situations; f) Select and use appropriate forms of rational numbers to solve real-world problems including those involving proportional relationships; g) Approximate, mentally and with calculators, the value of irrational numbers as they arise from problem situations; h) Express numbers in scientific notation (including negative exponents) in appropriate problem situations using a calculator; and i) Estimate answers and use formulas to solve application problems involving surface area and volume.

(2)   Strand: ALGEBRA - Standard: Students will understand algebraic concepts and applications.

(a)   5-8 Benchmark: Understand patterns, relations, and functions.

(i)   Grade 5 Performance Standards: a) Identify and graph ordered pairs in the first quadrant of the coordinate plane; b) Describe, represent, and analyze patterns and relationships; c) Identify, describe, and continue patterns presented in a variety of formats (e.g., numeric, visual, oral, written, kinesthetic, pictorial); and d) Generate a pattern using a written description.

(ii)   Grade 6 Performance Standards: a) Solve problems involving proportional relationships; b) Graph ordered pairs in the coordinate plane; c) Explain and use symbols to represent unknown quantities and variable relationships; d) Explain and use the relationships among ratios, proportions, and percents; and e) Make generalizations based on observed patterns and relationships.

(iii)   Grade 7 Performance Standards: a) Identify and continue patterns presented in a variety of formats; b) Represent a variety of relationships using tables, graphs, verbal rules, and possible symbolic notation, and recognize the same general pattern presented in different representations; c) Simplify numerical expressions by applying properties of rational numbers, and justify the process used; d) Interpret and evaluate expressions involving integer powers and simple roots; e) Graph and interpret linear functions; and f) Solve problems involving rate, average speed, distance, and time.

(iv)   Grade 8 Performance Standards: a) Move between numerical, tabular, and graphical representations of linear relationships; and b) Use variables to generalize patterns and information presented in tables, charts, and graphs: graph linear functions noting that the vertical change per unit of horizontal change (the slope of the graph) is always the same; plot the values of quantities whose ratios are always the same, fit a line to the plot, and understand that the slope of the line equals the quantities.

          (b)   5-8 Benchmark: Represent and analyze mathematical situations and structures using algebraic symbols.

              (i)   Grade 5 Performance Standards: a) Compute the value of the expression for specific numerical values of the variable; b) Use a letter to represent an unknown number; and c) Understand the differences between the symbols for "less than," "less than or equal to," "greater than," and "greater than or equal to."

              (ii)   Grade 6 Performance Standards: a) Solve problems involving proportional relationships; b) Use letters to represent an unknown in an equation; c) Solve one-step linear equations and inequalities in one variable with positive whole-number solutions; d) Demonstrate that a variable can represent a single quantity that changes; and e) Demonstrate how changes in one variable affect other variables.

              (iii)   Grade 7 Performance Standards: a) Write verbal expressions and sentences as algebraic expressions and equations: evaluate algebraic expressions; solve simple linear equations; graph and interpret results; b) Use variables and appropriate operations to write an expression, an equation, or an inequality that represents a verbal description; c) Use the order of operations to evaluate algebraic expressions; d) Simplify numerical expressions by applying properties of rational numbers; e) Graph linear functions and identify slope as positive or negative; and f) Use letters as variables in mathematical expressions to describe how one quantity changes when a related quantity changes.

              (iv)   Grade 8 Performance Standards: a) Demonstrate the difference between an equation and an expression; b) Solve two-step linear equations and inequalities in one variable with rational solutions; c) Evaluate formulas using substitution; d) Demonstrate understanding of the relationships between ratios, proportions, and percents and solve for a missing term in a proportion; e) Graph solution sets of linear equations in two variables on the coordinate plane; f) Formulate and solve problems involving simple linear relationships, find percents of a given number, variable situations, and unknown quantities; and g) Use symbols, variables, expressions, inequalities, equations, and simple systems of equations to represent problem situations that involve variables or unknown quantities.

          (c)   5-8 Benchmark: Use mathematical models to represent and understand quantitative relationships.

              (i)   Grade 5 Performance Standards: a) Use mathematical models to represent and explain mathematical concepts and procedures; b) Understand and use mathematical models such as: the number line to model the relationship between rational numbers and rational number operations; pictorial representation of addition and subtraction of rational numbers with regrouping; manipulatives or pictures to model computational procedures; graphs, tables, and charts to describe data; diagrams or pictures to model problem situations; and c) Demonstrate how a situation can be represented in more than one way.

              (ii)   Grade 6 Performance Standards: a) Develop and use mathematical models to represent and justify mathematical relationships found in a variety of situations; and b) Create, explain, and use mathematical models such as: Venn diagrams to show the relationships between the characteristics of two or more sets; equations and inequalities to model numerical relationships; three-dimensional geometric models; graphs, tables, and charts to interpret and analyze data.

              (iii)   Grade 7 Performance Standards: a) Create scale models and use them for dimensional drawings; b) Understand and use the coordinate plane to graph ordered pairs and linear equations; and c) Select and use an appropriate model for a particular situation.

              (iv)   Grade 8 Performance Standards: Generate different representations to model a specific numerical relationship given one representation of data (e.g., a table, a graph, an equation, a verbal description).

          (d)   5-8 Benchmark: Analyze changes in various contexts.

              (i)   Grade 5 Performance Standards: a) Recognize and create patterns of change from everyday life using numerical or pictorial representations; and b) Generalize patterns of change and recognize the same general patterns presented in different representations.

              (ii)   Grade 6 Performance Standards: a) Represent and explain changes using one-step equations with one variable; b) Solve problems that involve change using proportional relationships; c) Use ratios to predict changes in proportional situations; d) Use tables and symbols to represent and describe proportional and other relationships involving conversions, sequences, and perimeter; and e) Generate formulas to represent relationships involving changes in perimeter.

              (iii)   Grade 7 Performance Standards: a) Use variables and appropriate operations to write an expression, an equation, and/or an inequality that represents a verbal description involving change; b) Interpret and evaluate expressions involving integer powers and simple roots as they relate to change; c) Graph and interpret linear functions as they are used to solve problems; and d) Solve two-step equations and inequalities with

one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.

(iv)   Grade 8 Performance Standards: a) Use graphs, tables, and algebraic representations to make predictions and solve problems that involve change; b) Estimate, find, and justify solutions to problems that involve change using tables, graphs, and algebraic expressions; c) Use appropriate problem-solving strategies (e.g., drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table or graph, working a simpler problem, writing an algebraic expression or working backward) to solve problems that involve change; d) Solve multi-step problems that involve changes in rate, average speed, distance, and time; e) Analyze problems that involve change by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing, and observing patterns; f) Generalize a pattern of change using algebra and show the relationship among the equation, graph, and table of values; and g) Recognize the same general pattern of change presented in different representations.

(3)   Strand: GEOMETRY - Standard: Students will understand geometric concepts and applications.

(a)   5-8 Benchmark: Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematics arguments about geometric relationships.

(i)   Grade 5 Performance Standards: a) Identify, describe, and classify two-dimensional shapes and three-dimensional figures by their properties; b) Recognize and describe properties of regular polygons having up to ten sides; and c) Identify faces, edges, and bases on three-dimensional objects.

(ii)   Grade 6 Performance Standards: a) Identify, describe, and classify the properties of, and the relationships between, plane and solid geometric figures: measure, identify, and draw angles, perpendicular and parallel lines, rectangles, and triangles by using appropriate tools (e.g., straightedge, ruler, compass, protractor, drawing software); understand that the sum of angles of any triangle is 180 degrees and the sum of the angles of any quadrilateral is 360 degrees and use this information to solve problems; visualize and draw two-dimensional views of three-dimensional objects made from rectangular solids; b) Classify angles as right, obtuse, or straight; c) Describe the properties of geometric figures that include regular polygons, circles, ellipses, cylinders, cones, spheres, and cubes; d) Classify polygons as regular or irregular; e) Classify triangles as scalene, isosceles, or equilateral and by angles (i.e., right, acute, and obtuse); f) Identify angle, line, segment, and ray and use the symbols for each; and g) Describe the relationship between radius, diameter, and circumference of a circle.

(iii)   Grade 7 Performance Standards: a) Classify geometric figures as similar or congruent; b) Understand the concept of a constant (e.g., pi) and use the formulas for the circumference and area of a circle; c) Explain and use the Pythagorean theorem; d) Determine the radius, diameter, and circumference of a circle and explain their relationship; and e) Use properties to classify solids including pyramids, cones, prisms, and cylinders.

(iv)   Grade 8 Performance Standards: a) Recognize, classify, and discuss properties of all geometric figures including point, line, and plane; b) Identify arc, chord, and semicircle and explain their attributes; and c) Use the Pythagorean theorem and its converse to find the missing side of a right triangle and the lengths of the other line segments.

(b)   5-8 Benchmark: Specify locations and describe spatial relationships using coordinate geometry and other representational systems.

(i)   Grade 5 Performance Standards: Recognize perpendicular and parallel lines.

(ii)   Grade 6 Performance Standards: a) Use coordinate geometry to describe location on a plane; and b) Recognize skewed lines in space.

(iii)   Grade 7 Performance Standards: Construct and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine the image under translations and reflections.

(iv)   Grade 8 Performance Standards: Represent, formulate, and solve distance and geometry problems using the language and symbols of algebra and the coordinate plane and space (e.g., ordered triplets).

(c)   5-8 Benchmark: Apply transformations and use symmetry to analyze mathematical situations.

(i)   Grade 5 Performance Standards: Identify line of symmetry in simple geometric figures.

(ii)   Grade 6 Performance Standards: Identify line of symmetry with rotation and scaling.

(iii)   Grade 7 Performance Standards: Determine how perimeter and area are affected by changes of scale.

(iv)   Grade 8 Performance Standards: a) Describe the symmetry of three-dimensional figures; and b) Describe and perform single and multiple transformations that include rotation, reflection, translation, and dilation (i.e., shrink or magnify) to two-dimensional figures.

(d)   5-8 Benchmark: Use visualization, spatial reasoning, and geometric modeling to solve problems.

(i)   Grade 5 Performance Standards: a) Understand and compute the perimeter of regular polygons; and b) Identify and explain circumference, radius, and diameter.

(ii)   Grade 6 Performance Standards: Use appropriate technology, manipulatives, constructions, or drawings to recognize or compare geometric figures.

(iii)   Grade 7 Performance Standards: a) Compute the perimeter and area of common geometric shapes and use the results to find measures of less common objects; and b) Identify and describe the properties of two-dimensional figures: identify angles as vertical, adjacent, complementary, or supplementary and provide descriptions of these terms;
use the properties of complementary and supplementary angles and the sum of the angles of a triangle to solve problems involving an unknown angle; draw quadrilaterals and triangles from given information.

(iv)   Grade 8 Performance standards: a) Understand angle relationships formed by parallel lines cut by a transversal; b) Recognize and apply properties of corresponding parts of similar and congruent triangles and quadrilaterals; c) Represent and solve problems relating to size, shape, area, and volume using geometric models; d) Develop and use formulas for area, perimeter, circumference, and volume; and e) Construct two-dimensional patterns for three-dimensional models (e.g., cylinders, prisms, cones).

(4)   Strand: MEASUREMENT - Standard: Students will understand measurement systems and applications.

(a)   5-8 Benchmark: Understand measurable attributes of objects and the units, systems, and processes of measurement.

(i)   Grade 5 Performance Standards: a) Understand properties (e.g., length, area, weight, volume) and select the appropriate type of unit for measuring each using both U.S. customary and metric systems; b) Select and use appropriate units and tools to measure according to the degree of accuracy required in a particular problem-solving situation; c) Solve problems involving linear measurement, weight, and capacity (e.g., measuring to the nearest sixteenth of an inch or nearest millimeter; using ounces, milliliters, or pounds and kilograms) to the appropriate degree of accuracy; and d) Perform one-step conversions within a system of measurement (e.g., inches to feet, centimeters to meters).

(ii)   Grade 6 Performance Standards: a) Perform multi-step conversions of measurement units to equivalent units within a given system (e.g., 36 inches equals 3 feet or 1 yard); b) Estimate measurement in both U.S. customary and metric units; c) Select and use units of appropriate size and type to measure angles (e.g., degrees, radians), perimeter, area, and capacity in both U.S. customary and metric systems; and d) Use standard units of linear measurement to the nearest sixteenth of an inch; metric measurements to the nearest millimeter.

(iii)   Grade 7 Performance Standards: a) Choose appropriate units of measure and ratios to recognize new equivalences (e.g., 1 square yard equals 9 square feet) to solve problems; b) Select and use the appropriate size and type of unit for a given measurement situation; c) Compare masses, weights, capacities, geometric measures, times, and temperatures within measurement systems; d) Approximate the relationship between standard and metric measurement systems (e.g., inches and centimeters, pounds and kilograms, quarts and liters); and e) Use measures expressed as rates and measures expressed as products to solve problems, check the units of the solutions, and analyze the reasonableness of the answer.

(iv)   Grade 8 Performance Standards: a) Understand the concept of volume and use the appropriate units in common measuring systems (e.g., cubic centimeter, cubic inch, cubic yard) to compute the volume of rectangular solids; and b) Use changes in measurement units (e.g., square inches, cubic feet) to perform conversions from one-, two-, and three-dimensional shapes.

(b)   5-8 Benchmark: Apply appropriate techniques, tools, and formulas to determine measurements.

(i)   Grade 5 Performance Standards: a) Solve measurement problems using appropriate tools involving length, perimeter, weight, capacity, time, and temperature; b) Select and use strategies to estimate measurements including length, distance, capacity, and time; and c) Apply strategies and use tools for estimating and measuring the perimeter of regular and irregular shapes.

(ii)   Grade 6 Performance Standards: a) Apply various measurement techniques and tools, units of measure, and degrees of accuracy to find accurate rational number representations for length, liquid, weight, perimeter, temperature, and time; b) Select and use formulas for perimeters of squares and rectangles; c)

Select and use strategies to estimate measurements including angle measure and capacity; and d) Select and justify the selection of measurement tools, units of measure, and degrees of accuracy appropriate to the given situation.

    (iii) Grade 7 Performance Standards: a) Apply strategies and formulas to find missing angle measurements in triangles and quadrilaterals; b) Select and use formulas to determine the circumference of circles and the area of triangles, parallelograms, trapezoids, and circles; and c) Solve problems involving scale factors, ratios, and proportions.

    (iv) Grade 8 Performance Standards: a) Use ratios and proportions to measure hard-to-measure objects; b) Use estimation to solve problems; c) Use proportional relationships in similar shapes to find missing measurements; d) Apply strategies to determine the surface area and volume of prisms, pyramids, and cylinders; e) Perform conversions with multiple terms between metric and U.S. standard measurement systems; f) Estimate volume in cubic units; and g) Solve simple problems involving rates and derived measurements for such properties as velocity and density.

   (5) Strand: DATA ANALYSIS AND PROBABILITY - Standard: Students will understand how formulate questions, analyze data, and determine probabilities.

    (a) 5-8 Benchmark: Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.

    (i) Grade 5 Performance Standards: a) Construct, read, analyze, and interpret tables, charts, graphs, and data plots; b) Construct, interpret, and analyze data from graphical representations and draw simple conclusions using bar graphs, line graphs, circle graphs, frequency tables, and Venn diagrams; c) Display, analyze, compare, and interpret different data sets, including data sets of different sizes; d) Organize and display single-variable data in appropriate graphs and representations; e) Organize, read, and display numerical (quantitative) and non-numerical (qualitative) data in a clear, organized, and accurate manner including correct titles, labels, and intervals or categories including: frequency tables; stem and leaf plots; bar, line, and circle graphs; Venn diagrams; pictorial displays; charts and tables; and f) Formulate questions and identify data to be collected to correctly answer a question.

    (ii) Grade 6 Performance Standards: a) Use statistical representations to analyze data; b) Draw and compare different graphical representations of the same data; c) Use mean, median, mode, and range to describe data; d) Sketch circle graphs to display data; e) Solve problems by collecting, organizing, displaying and interpreting data; f) Compare different samples of a population with the entire population and determine the appropriateness of using a sample; g) Conduct and explain sampling techniques such as observations, surveys, and random sampling for gathering data; h) Determine the median for a rational number data set containing an odd number of data points; i) Calculate and explain the median for a whole number data set containing an even number of data points; j) Explain advantages and disadvantages of using various display formats for a specific data set; and k) Formulate and solve problems by collecting, organizing, displaying, and interpreting data.

    (iii) Grade 7 Performance Standards: a) Describe how data representations influence interpretation; b) Select and use appropriate representation for presenting collected data and justify the selection; c) Use measures of central tendency and spread to describe a set of data; d) Choose between median and mode to describe a set of data and justify the choice for a particular situation; e) Determine the quartiles of a data set; f) Identify ordered pairs of data from a graph and interpret the data in terms of the situation depicted by the graph; g) Use various scales and formats to display the same data set; h) Identify and explain the misleading representations of data; i) Collect, organize, and represent data sets that have one or more variables and identify relationships among variables within a data set; j) Compute the minimum, lower quartile, median, upper quartile, and maximum of a data set; k) Identify and explain the effects of scale and/or interval changes on graphs of whole number data sets; l) Use and explain sampling techniques (e.g., observations, surveys, and random sampling) for gathering data; and m) Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, and selecting, collecting, and displaying appropriate data to address the problem.

    (iv) Grade 8 Performance Standards: a) Represent two numerical variables on a plot, describe how the data points are distributed, and identify relationships that exist between the two variables; b) Generate, organize, and interpret real numbers in a variety of situations; c) Organize, analyze, and display appropriate quantitative and qualitative data to address specific questions including: frequency distributions; plots; histograms; bar, line, and pie graphs; diagram and pictorial displays; charts and tables; d) Select the appropriate measure of central tendency to describe a set of data for a particular problem situation; e) Simulate an event selecting and using different models; and f) Develop an appropriate strategy using a variety of data from surveys, samplings, estimations, and inferences to address a specific problem.

    (b) 5-8 Benchmark: Select and use appropriate statistical methods to analyze data.

(i)   Grade 5 Performance Standards: a) Organize and display single-variable data in appropriate graphs and representations and determine which types of graphs are appropriate for various data sets; b) Use fractions and percentages to compare data sets of different sizes; and c) Correctly rank the values of a numerical data set containing simple fractions and decimals, identify maximum and minimum data values, and calculate the range for a data set.

(ii)   Grade 6 Performance Standards: a) Choose an appropriate graphical format to organize and represent data; b) Describe the effects of missing or incorrect data; c) Compute and analyze statistical measurements for data sets: understand how additional data added to data sets may affect the computations of central tendency; understand how the inclusion or exclusion of outliers affects measures of central tendency; know why a specific measure of central tendency provides the most useful information in a given context; d) Use data samples of a population and describe the characteristics and limitations of the sample; e) Identify different ways of selecting a sample (e.g., convenience sampling, responses to a survey, random sampling) and which method makes a sample more representative for a population; f) Explain how the way a question is asked in a survey might influence the results obtained; g) Identify data that represent sampling errors and explain why the sample and the display might be biased; and h) Identify claims based on statistical data and, in sample cases, evaluate the validity and usefulness of the claims.

(iii)   Grade 7 Performance Standards: a) Choose and justify appropriate measures of central tendencies (e.g., mean, median, mode, range) to describe given or derived data; b) Know various ways to display data sets (e.g., stem and leaf plot, box and whisker plot, scatter plots) and use these forms to display a single set of data or to compare two sets of data; c) Use the analysis of data to make convincing arguments; d) Use appropriate technology to gather and display data sets and identify the relationships that exist among variables within the data set; e) Use data samples of a population and describe the characteristics and limitations of the sample; f) Identify data that represent sampling errors and explain why the sample and the display might be biased; and g) Identify claims based on statistical data and evaluate the validity of the claims.

(iv)   Grade 8 Performance Standards: a) Use changes in scales, intervals, or categories to help support a particular interpretation of data; b) Generate, organize, and interpret real number and other data in a variety of situations; c) Analyze data to make decisions and to develop convincing arguments from data displayed in a variety of formats that include: plots; distributions; graphs; scatter plots; diagrams; pictorial displays; charts and tables; Venn diagrams; d) Interpret and analyze data from graphical representations and draw simple conclusions (e.g., line of best fit); e) Evaluate and defend the reasonableness of conclusions drawn from data analysis; f) Use appropriate central tendency and spread as a means for effective decision-making in analyzing data and outliers; g) Identify simple graphic misrepresentations and distortions of sets of data (e.g., unequal interval sizes, omission of parts of axis range, scaling); and h) Use appropriate technology to display data as lists, tables, matrices, graphs, and plots and to analyze the relationships of variables in the data displayed.

(c)   5-8 Benchmark: Develop and evaluate inferences and predictions that are based on data.

(i)   Grade 5 Performances Standards: a) Make and justify valid inferences, predictions, and arguments based on statistical analysis; b) Compare a given prediction with the results of an investigation; c) Use counting strategies to determine all the possible outcomes of a particular familiar event; d) Find all possible outcome sets involving four or more sets of objects; e) Evaluate the reasonableness of inferences that are based on data in the context of the original solution; f) Identify the method used to make an inference and/or a prediction on a given data set and solve similar problems; g) Determine the accuracy of a prediction or an inference based on the accuracy of the data in a given data set; and h) List all possible outcomes of simple events.

(ii)   Grade 6 Performance Standards: a) Identify claims based on statistical data and evaluate the validity of the claim; b) Conduct observations, surveys, experiments and/or simulations, record the results in charts, tables, or graphs, and use the results to draw conclusions and make predictions; c) Find all possible combinations in a given set (e.g., the number of ways a set of books can be arranged on a shelf); and d) Compare expected results with actual results in a simple experiment.

(iii)   Grade 7 Performance Standards: a) Formulate and justify mathematical conjectures based on data and a general description of the mathematical question or problem posed; b) Analyze data to make accurate inferences, predictions, and to develop convincing arguments from data displayed in a variety of forms; and c) Approximate a line of best fit for a data set in a scatter plot form and make predictions using the simple equation of that line.

(iv)   Grade 8 Performance Standards: a) Describe how changes in scale, intervals, or categories influence arguments for a particular interpretation of the data; b) Describe how reader bias, measurement errors, and display distortion can affect the interpretation of data, predictions, and inferences based on data; c) Conduct simple experiments and/or simulations, record results in charts, tables, or graphs, and use the results to

draw conclusions and make predictions; and d) Compare expected results with experimental results and information used in predictions and inferences.

    (d)   5-8 Benchmark: Understand and apply basic concepts of probability.

    (i)   Grade 5 Performance Standards: a) Determine probabilities through experiments and/or simulations and compare the results with mathematical expressions; b) Make predictions from the results of student-generated experiments of single events; c) Identify simple experiments where the probabilities of all outcomes are equal; d) Describe and predict the results of a probability experiment; e) Use fractions to describe the results of an experiment; and f) Use probability to generalize from a simple pattern or set of examples and justify why the generalization is reasonable.

    (ii)   Grade 6 Performance Standards: a) List all possible outcomes for a compound event composed of two independent events and recognize whether an outcome is certain, impossible, likely, or unlikely; b) Determine and compare experimental (empirical) and mathematical (theoretical) probabilities (e.g., flipping two color counters); c) Determine theoretical and experimental probabilities and use them to make predictions about events; d) Represent all possible outcomes for compound events in an organized way (e.g., tables, grids, tree diagrams) and express the theoretical probability of each outcome; e) Use data to estimate the probability of future events (e.g., batting averages); f) Represent probabilities as ratios, proportions, decimals between 0 and 1, and percentages between 0 and 100 and verify that the probabilities computed are reasonable; know that if P is the probability of an event, 1- P is the probability of the event not occurring; and g) Describe the difference between independent and dependent events and identify situations involving independent or dependent events.

    (iii)   Grade 7 Performance Standards: a) Determine the probability of a compound event composed of two independent events; b) Identify examples of events having the probability of one or zero; c) Describe the probability of events using fractions, decimals, and percents; d) Express probability as a fraction, zero, or one; e) Use probability to generate convincing arguments, draw conclusions, and make decisions in a variety of situations; f) Make predictions based on theoretical probabilities of compound events; and g) Determine the probability of a simple event or a compound event composed of simple, independent events.

    (iv)   Grade 8 Performance Standards: a) Calculate the odds of a desired outcome in a simple experiment; b) Design and use an appropriate simulation to estimate the probability of a real-world event (e.g., disk toss, cube toss); c) Explain the relationship between probability and odds and calculate the odds of a desired outcome in a simple experiment; d) Use theoretical or experimental probability to make predictions about real-world events; e) Use probability to generate convincing arguments, draw conclusions, and make decisions in a variety of situations; and f) Understand that the probability of two unrelated events occurring is the sum of the two individual possibilities and that the probability of one event following another, in independent trials, is the product of the two probabilities.

    C.   9-12 MATHEMATICS

    (1)   Strand: ALGEBRA, FUNCTIONS, AND GRAPHS - Standard: Students will understand algebraic concepts and applications.

    (a)   9-12 Benchmark: Represent and analyze mathematical situations and structures using algebraic symbols.

    (i)   Grade 9-12 Performance Standards: Classify numbers and members of the following sets: natural; whole; integers
rationals; irrationals.

    (ii)   Grade 9-12 Performance Standards: Simplify numerical expressions using the order of operations, including exponents.

    (iii)   Grade 9-12 Performance Standards: Evaluate the numerical value of expressions of one or more variables that are: polynomial; rational; radical.

    (iv)   Grade 9-12 Performance Standards: Simplify algebraic monomial expressions raised to a power (e.g., $[5xy^2]^3$) and algebraic binomial (e.g., $[5x^2 + y]^2$) expressions raised to a power.

    (v)   Grade 9-12 Performance Standards: Compare and order polynomial expressions by degree.

    (vi)   Grade 9-12 Performance Standards: Represent and analyze relationships using written and verbal expressions, tables, equations, and graphs, and describe the connections among those representations: translate from verbal expression to algebraic formulae (e.g., "Set up the equations that represent the data in the following equation: John's father is 23 years older than John. John is 4 years older than his sister Jane. John's mother is 3 years younger than John's father. John's mother is 9 times as old as Jane. How old are John, Jane, John's mother, and John's father?"); given data in a table, construct a function that represents these data (linear only); given a graph, construct a function that represents the graph (linear only).

            (vii)   Grade 9-12 Performance Standards: Know, explain, and use equivalent representations for the same real number including: integers; decimals; percents; ratios; scientific notation; numbers with integer exponents; inverses (reciprocal); prime factoring.

            (viii)   Grade 9-12 Performance Standards: Simplify algebraic expressions using the distributive property.

            (ix)   Grade 9-12 Performance Standards: Explain and use the concept of absolute value.

            (x)   Grade 9-12 Performance Standards: Know, explain, and use equivalent representations for algebraic expressions.

            (xi)   Grade 9-12 Performance Standards: Simplify square roots and cube roots with monomial radicands that are perfect squares or perfect cubes (e.g., $9a^2x^4$).

            (xii)   Grade 9-12 Performance Standards: Calculate powers and roots of real numbers, both rational and irrational.

            (xiii)   Grade 9-12 Performance Standards: Solve: formulas for specified variables; radical equations involving one radical.

            (xiv)   Grade 9-12 Performance Standards: Factor polynomials, difference of squares and perfect square trinomials, and the sum and difference of cubes.

            (xv)   Grade 9-12 Performance Standards: Simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.

            (xvi)   Grade 9-12 Performance Standards: Manipulate simple expressions with + and – exponents.

            (xvii)   Grade 9-12 Performance Standards: Use the four basic operations (+, -, x, ÷) with: linear expressions; polynomial expressions; rational expressions.

        (b)   9-12 Benchmark: Understand patterns, relations, functions, and graphs.

            (i)   Grade 9-12 Performance Standards: Distinguish between the concept of a relation and a function.

            (ii)   Grade 9-12 Performance Standards: Determine whether a relation defined by a graph, a set of ordered pairs, a table of values, an equation, or a rule is a function.

            (iii)   Grade 9-12 Performance Standards: Describe the concept of a graph of a function.

            (iv)   Grade 9-12 Performance Standards: Translate among tabular, symbolic, and graphical representations of functions.

            (v)   Grade 9-12 Performance Standards: Explain and use function notation.

            (vi)   Grade 9-12 Performance Standards: Determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.

            (vii)   Grade 9-12 Performance Standards: Identify the independent and dependent variables from an application problem (e.g., height of a child).

            (viii)   Grade 9-12 Performance Standards: Describe the concept of a graph of an equation.

            (ix)   Grade 9-12 Performance Standards: Understand symmetry of graphs.

            (x)   Grade 9-12 Performance Standards: Analyze and describe middle and end (asymptotic) behavior of linear, quadratic, and exponential functions, and sketch the graphs of functions.

            (xi)   Grade 9-12 Performance Standards: Work with composition of functions (e.g., find f of g when f(x) = 2x - 3 and g(x) = 3x - 2), and find the domain, range, intercepts, zeros, and local maxima or minima of the final function.

            (xii)   Grade 9-12 Performance Standards: Use the quadratic formula and factoring techniques to determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points.

            (xiii)   Grade 9-12 Performance Standards: Apply quadratic equations to physical phenomena (e.g., the motion of an object under the force of gravity).

        (c)   9-12 Benchmark: Use mathematical models to represent and understand quantitative relationships.

            (i)   Grade 9-12 Performance Standards: Model real-world phenomena using linear and quadratic equations and linear inequalities (e.g., apply algebraic techniques to solve rate problems, work problems, and percent mixture problems; solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest; apply quadratic equations to model throwing a baseball in the air).

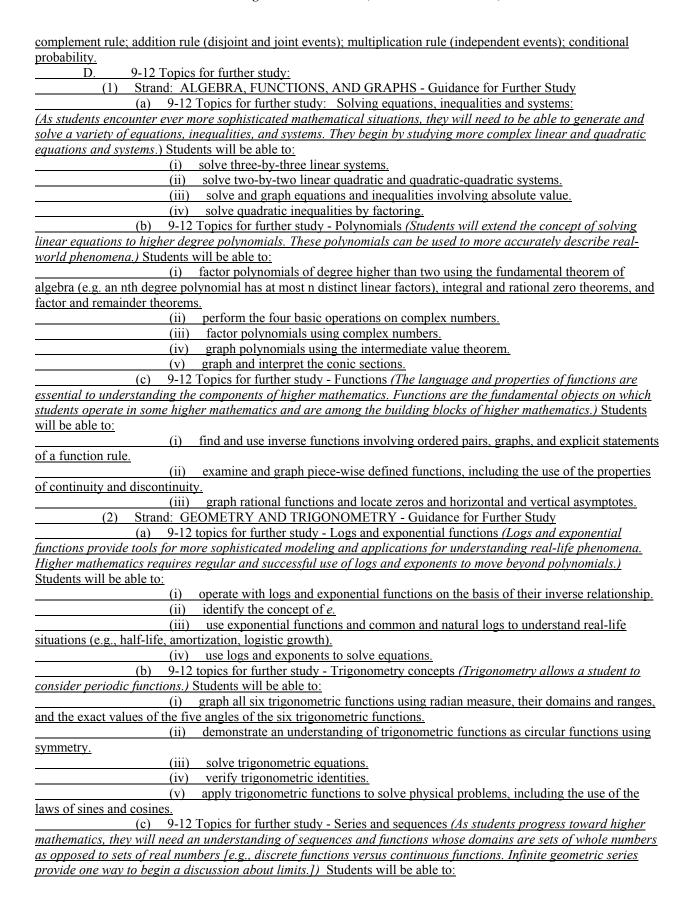   (ii)   Grade 9-12 Performance Standards: Use a variety of computational methods (e.g., mental arithmetic, paper and pencil, technological tools).

(iii)   Grade 9-12 Performance Standards: Express the relationship between two variables using a table with a finite set of values and graph the relationship.

(iv)   Grade 9-12 Performance Standards: Express the relationship between two variables using an equation and a graph: graph a linear equation and linear inequality in two variables; solve linear inequalities and equations in one variable; solve systems of linear equations in two variables and graph the solutions; use the graph of a system of equations in two variables to help determine the solution.

(v)   Grade 9-12 Performance Standards: Solve applications involving systems of equations.

(vi)   Grade 9-12 Performance Standards: Evaluate numerical and algebraic absolute value expressions.

(vii)   Grade 9-12 Performance Standards: Create a linear equation from a table of values containing co-linear data.

(viii)   Grade 9-12 Performance Standards: Determine the solution to a system of equations in two variables from a given graph.

(ix)   Grade 9-12 Performance Standards: Generate an algebraic sentence to model real-life situations.

(x)   Grade 9-12 Performance Standards: Write an equation of the line that passes through two given points.

(xi)   Grade 9-12 Performance Standards: Understand and use: such operations as taking the inverse, finding the reciprocal, taking a root, and raising to a fractional power; the rules of exponents.

(xii)   Grade 9-12 Performance Standards: Verify that a point lies on a line, given an equation of the line, and be able to derive linear equations by using the point-slope formula.

(d)   9-12 Benchmark: Analyze changes in various contexts.

(i)   Grade 9-12 Performance Standards: Analyze the effects of parameter changes on these functions: linear (e.g., changes in slope or coefficients); quadratic (e.g., f[x-a] changes coefficients and constants); exponential (e.g., changes caused by increasing $x[x + c]$ or $[a^x]$); polynomial (e.g., changes caused by positive or negative values of a, or in a constant c).

(ii)   Grade 9-12 Performance Standards: Solve routine two- and three-step problems relating to change using concepts such as: exponents; factoring; ratio; proportion; average; percent.

(iii)   Grade 9-12 Performance Standards: Calculate the percentage of increase and decrease of a quantity.

(iv)   Grade 9-12 Performance Standards: Analyze the general shape of polynomial expressions and equations for different degree polynomials (e.g., positive and negative general shapes for third-, fourth-, and fifth-degree polynomials).

(v)   Grade 9-12 Performance Standards: Estimate the rate of change of a function or equation by finding the slope between two points on the graph.

(vi)   Grade 9-12 Performance Standards: Evaluate the estimated rate of change in the context of the problem.

(vii)   Grade 9-12 Performance Standards: Know Pascal's triangle and use it to expand binomial expressions that are raised to positive integer powers.

(2)   Strand: GEOMETRY AND TRIGONOMETRY - Standard: Students will understand geometric concepts and applications.

(a)   9-12 Benchmark: Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

(i)   Grade 9-12 Performance Standards: Interpret and draw two-dimensional objects and find the area and perimeter of basic figures (e.g., rectangles, circles, triangles, other polygons [e.g., rhombi, parallelograms, trapezoids]).

(ii)   Grade 9-12 Performance Standards: Find the area and perimeter of a geometric figure composed of a combination of two or more rectangles, triangles, and/or semicircles with just edges in common.

(iii)   Grade 9-12 Performance Standards: Find and use measures of sides and interior and exterior angles of triangles and polygons to classify figures (e.g., scalene, isosceles, and equilateral triangles; rectangles [square and non-square]; other convex polygons).

(iv)   Grade 9-12 Performance Standards: Interpret and draw three-dimensional objects and find the surface area and volume of basic figures (e.g., spheres, rectangular solids, prisms, polygonal cones), and

calculate the surface areas and volumes of these figures as well as figures constructed from unions of rectangular solids and prisms with faces in common, given the formulas for these figures.

          (v)   Grade 9-12 Performance Standards: Demonstrate an understanding of simple aspects of a logical argument: identify the hypothesis and conclusion in logical deduction; use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.

          (vi)   Grade 9-12 Performance Standards: Demonstrate an understanding of inductive and deductive reasoning, explain the difference between inductive and deductive reasoning, and identify and provide examples of each: for inductive reasoning, demonstrate understanding that showing a statement is true for a finite number of examples does not show it is true for all cases unless the cases verified are all cases; for deductive reasoning, prove simple theorems.

          (vii)   Grade 9-12 Performance Standards: Write geometric proofs (including proofs by contradiction), including: theorems involving the properties of parallel lines cut by a transversal line and the properties of quadrilaterals; theorems involving complementary, supplementary, and congruent angles; theorems involving congruence and similarity; the Pythagorean theorem (tangram proof).

          (b)   9-12 Benchmark: Specify locations and describe spatial relationships using coordinate geometry and other representational systems.

          (i)   Grade 9-12 Performance Standards: Demonstrate understanding of the construction of the coordinate plane, know the names of the origin, coordinate axes and four quadrants, draw and label them correctly, find the coordinates of an indicated point, and plot a point with given coordinates.

          (ii)   Grade 9-12 Performance Standards: Determine the midpoint and distance between two points within a coordinate system and relate these ideas to geometric figures in the plane (e.g., find the center of a circle given two endpoints of a diameter of the circle).

          (iii)   Grade 9-12 Performance Standards: Given two linear equations, determine whether the lines are parallel, perpendicular, or coincide.

          (iv)   Grade 9-12 Performance Standards: Use basic geometric ideas (e.g., the Pythagorean theorem, area, and perimeter of objects) in the context of the Euclidean Plane, calculate the perimeter of a rectangle with integer coordinates and sides parallel to the coordinate axes and with sides not parallel.

          (c)   9-12 Benchmark: Apply transformations and use symmetry to analyze mathematical situations.

          (i)   Grade 9-12 Performance Standards: Describe the effect of rigid motions on figures in the coordinate plane and space that include rotations, translations, and reflections:
determine whether a given pair of figures on a coordinate plane represents the effect of a translation, reflection, rotation, and/or dilation; sketch the planar figure that is the result of a given transformation of this type.

          (ii)   Grade 9-12 Performance Standards: Deduce properties of figures using transformations that include translations, rotations, reflections, and dilations in a coordinate system:
 identify congruency and similarity in terms of transformations; determine the effects of the above transformations on linear and area measurements of the original planar figure.

          (d)   9-12 Benchmark: Use visualization, spatial reasoning, and geometric modeling to solve problems.

          (i)   Grade 9-12 Performance Standards: Solve real-world problems using congruence and similarity relationships of triangles (e.g., find the height of a pole given the length of its shadow).

          (ii)   Grade 9-12 Performance Standards: Solve problems involving complementary, supplementary, and congruent angles.

          (iii)   Grade 9-12 Performance Standards: Solve problems involving the perimeter, circumference, area, volume, and surface area of common geometric figures (e.g., "Determine the surface area of a can of height h and radius r. How does the surface area change when the height is changed to 3h? How does the surface area change when the radius is changed to 3r? How does the surface area change when both h and r are doubled?").

          (iv)   Grade 9-12 Performance Standards: Solve problems using the Pythagorean theorem (e.g., "Given the length of a ladder and the distance of the base of the ladder from a wall, determine the distance up the wall to the top of the ladder").

          (v)   Grade 9-12 Performance Standards: Understand and use elementary relationships of basic trigonometric functions defined by the angles of a right triangle (e.g., "What is the radius of a circle with an inscribed regular octagon with the length of each side being exactly 2 feet?").

          (vi)   Grade 9-12 Performance Standards: Use trigonometric functions to solve for the length of the second leg of a right triangle given the angles and the length of the first leg. (e.g., "A surveyor

determines that the angle subtended by a two-foot stick at right angles to his transit is exactly one degree. What is the distance from the transit to the base of the measuring stick?").

            (vii)   Grade 9-12 Performance Standards: Know and use angle and side relationships in problems with special right triangles (e.g., 30-, 45-, 60-, and 90-degree triangles).

        (3)   Strand: DATA ANALYSIS AND PROBABILITY - Standard: Students will understand how to formulate questions, analyze data, and determine probabilities.

          (a)   9-12 Benchmark: Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.

            (i)   Grade 9-12 Performance Standards: Understand the differences between the various methods of data collection.

            (ii)   Grade 9-12 Performance Standards: Know the characteristics of a well-designed and well-conducted survey: differentiate between sampling and census; differentiate between a biased and an unbiased sample.

            (iii)   Grade 9-12 Performance Standards: Know the characteristics of a well-designed and well-conducted experiment: differentiate between an experiment and an observational study; recognize sources of bias in poorly designed experiments.

            (iv)   Grade 9-12 Performance Standards: Understand the role of randomization in well-designed surveys and experiments.

          (b)   9-12 Benchmark: Select and use appropriate statistical methods to analyze data.

            (i)   Grade 9-12 Performance Standards: Understand the meaning of measurement data and categorical data, and of the term "variable."

            (ii)   Grade 9-12 Performance Standards: Understand the meaning of "univariate" (i.e., one variable) and "bivariate" (i.e., two variable) data.

            (iii)   Grade 9-12 Performance Standards: For univariate data, be able to display the distribution and describe its shape using appropriate summary statistics, and understand the distinction between a statistic and a parameter: construct and interpret frequency tables, histograms, stem and leaf plots, and box and whisker plots; calculate and apply measures of central tendency (mean, median, and mode) and measures of variability (range, quartiles, standard deviation); compare distributions of univariate data using back-to-back stem and leaf plots and parallel box and whisker plots.

            (iv)   Grade 9-12 Performance Standards: For bivariate data, be able to display a scatter plot and describe its shape: fit a linear model to a set of data using technological tools; describe and interpret the relationship/correlation between two variables using technological tools.

          (c)   9-12 Benchmark: Develop and evaluate inferences and predictions that are based on data.

            (i)   Grade 9-12 Performance Standards: Compare and draw conclusions between two or more sets of univariate data using basic data analysis techniques and summary statistics.

            (ii)   Grade 9-12 Performance Standards: Draw conclusions concerning the relationships among bivariate data: make predictions from a linear pattern in data; determine the strength of the relationship between two sets of data by examining the correlation; understand that correlation does not imply a cause-and-effect relationship.

            (iii)   Grade 9-12 Performance Standards: Use simulations to explore the variability of sample statistics from a known population and construct sampling distributions.

            (iv)   Grade 9-12 Performance Standards: Understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference.

            (v)   Grade 9-12 Performance Standards: Evaluate published reports that are based on data by examining the design of the study, the appropriateness of the data analysis, and the validity of conclusions.

          (d)   9-12 Benchmark: Understand and apply basic concepts of probability.

            (i)   Grade 9-12 Performance Standards: Explain the concept of a random variable.

            (ii)   Grade 9-12 Performance Standards: Understand the concept of probability as relative frequency.

            (iii)   Grade 9-12 Performance Standards: Use simulations to compute the expected value and probabilities of random variables in simple cases.

            (iv)   Grade 9-12 Performance Standards: Distinguish between independent and dependent events.

            (v)   Grade 9-12 Performance Standards: Understand how to compute the probability of an event using the basic rules of probability:

complement rule; addition rule (disjoint and joint events); multiplication rule (independent events); conditional probability.

        D.   9-12 Topics for further study:

          (1)   Strand: ALGEBRA, FUNCTIONS, AND GRAPHS - Guidance for Further Study

            (a)   9-12 Topics for further study: Solving equations, inequalities and systems: *(As students encounter ever more sophisticated mathematical situations, they will need to be able to generate and solve a variety of equations, inequalities, and systems. They begin by studying more complex linear and quadratic equations and systems*.) Students will be able to:

                (i)   solve three-by-three linear systems.

                (ii)   solve two-by-two linear quadratic and quadratic-quadratic systems.

                (iii)   solve and graph equations and inequalities involving absolute value.

                (iv)   solve quadratic inequalities by factoring.

            (b)   9-12 Topics for further study - Polynomials *(Students will extend the concept of solving linear equations to higher degree polynomials. These polynomials can be used to more accurately describe real-world phenomena.)* Students will be able to:

                (i)   factor polynomials of degree higher than two using the fundamental theorem of algebra (e.g. an nth degree polynomial has at most n distinct linear factors), integral and rational zero theorems, and factor and remainder theorems.

                (ii)   perform the four basic operations on complex numbers.

                (iii)   factor polynomials using complex numbers.

                (iv)   graph polynomials using the intermediate value theorem.

                (v)   graph and interpret the conic sections.

            (c)   9-12 Topics for further study - Functions *(The language and properties of functions are essential to understanding the components of higher mathematics. Functions are the fundamental objects on which students operate in some higher mathematics and are among the building blocks of higher mathematics.)* Students will be able to:

                (i)   find and use inverse functions involving ordered pairs, graphs, and explicit statements of a function rule.

                (ii)   examine and graph piece-wise defined functions, including the use of the properties of continuity and discontinuity.

                (iii)   graph rational functions and locate zeros and horizontal and vertical asymptotes.

          (2)   Strand: GEOMETRY AND TRIGONOMETRY - Guidance for Further Study

            (a)   9-12 topics for further study - Logs and exponential functions *(Logs and exponential functions provide tools for more sophisticated modeling and applications for understanding real-life phenomena. Higher mathematics requires regular and successful use of logs and exponents to move beyond polynomials.)* Students will be able to:

                (i)   operate with logs and exponential functions on the basis of their inverse relationship.

                (ii)   identify the concept of *e.*

                (iii)   use exponential functions and common and natural logs to understand real-life situations (e.g., half-life, amortization, logistic growth).

                (iv)   use logs and exponents to solve equations.

            (b)   9-12 topics for further study - Trigonometry concepts *(Trigonometry allows a student to consider periodic functions.)* Students will be able to:

                (i)   graph all six trigonometric functions using radian measure, their domains and ranges, and the exact values of the five angles of the six trigonometric functions.

                (ii)   demonstrate an understanding of trigonometric functions as circular functions using symmetry.

                (iii)   solve trigonometric equations.

                (iv)   verify trigonometric identities.

                (v)   apply trigonometric functions to solve physical problems, including the use of the laws of sines and cosines.

            (c)   9-12 Topics for further study - Series and sequences *(As students progress toward higher mathematics, they will need an understanding of sequences and functions whose domains are sets of whole numbers as opposed to sets of real numbers [e.g., discrete functions versus continuous functions. Infinite geometric series provide one way to begin a discussion about limits.])* Students will be able to:

               (i)   use algebraic techniques to generate the specific formulas for arithmetic and geometric sequences and series.

               (ii)   extend the concept of series to infinite geometric series.

               (iii)   use the language and notation of limits.

               (iv)   use mathematical induction to prove various mathematical statements.

[10-31-96; 6.30.2.14 NMAC - Rn, 6 NMAC 3.2.11, 11-14-00; A, 03-14-03]